*International Journal of Precious Engineering Research and Applications (IJPERA)*
*ISSN (Online): 2456-2734*
*www.ijpera.com Volume 2 Issue 5 ǁ November 2017 ǁ PP. 23-30*

# Lyapunov-Type Inequalities For A Fractional $q$- Difference Equation Involving $p$- Laplacian Operator

[1]Yiding Liu [1*]Chengmin Hou
*[1]Department of Mathematics, Yanbian University, Yanji, 133002, P.R. China*
*Corresponding Author: [1]Yiding Liu*

**ABSTRACT:** In this paper, we present new Lyapunov-type inequalities for a boundary value problem of fractional $q$- difference equation with $p$- Laplacian operator. The obtained inequalities are used to obtain a lower bound for the eigenvalues of corresponding equations.

**Keywords:** Lyapunov-type inequality, fractional $q$- derivative, eigenvalues, Boundary Value Problem.

Date of Submission: 01-12-2017 Date of acceptance: 09-12-2017

## I. INTRODUCTION

The $p$- Laplacian operator arises in different mathematical models that describe physicaland natural phenomena (see, for example, [1-]).

In this paper, we present some Lyapunov-type inequalities for a fractional $q$- difference equation with $p$- Laplacian operator. More precisely, we are interested with the nonlinear fractional boundary value problem

$$\begin{cases} D_{q,a^+}^{\beta}\left(\Phi_p\left(D_{q,a^+}^{\alpha}u(t)\right)\right)+\chi(t)\Phi_p u(t)=0, & a<t<b \\ u(a)=D_q u(a)=D_q u(b)=0, & D_{q,a^+}^{\alpha}u(a)=D_{q,a^+}^{\alpha}u(b)=0 \end{cases} \quad (1.1)$$

where $2<\alpha\le 3,\ 1<\beta\le 2,\ D_{q,a^+}^{\alpha},\ D_{q,a^+}^{\beta}$ arethe Riemann-Liouville fractional $q$- derivatives oforders $\alpha,\beta,\Phi_p(s)=|s|^{p-2}s,\ p>1,$ and $\chi:[a,b]\to\mathbb{R}$ is a continuous function.

Under certain assumptions imposed on the function $g$, we obtain necessary conditions for the existence of nontrivial solutions to $(1.1)$. Some applications to eigenvalue problems are also presented.

For completeness, let us recall the standard Lyapunov inequality [5], which states thatif $u$ is a nontrivial solution of the problem

$$\begin{cases} u''(t)+\chi(t)u(t)=0, & a<t<b \\ u(a)=u(b)=0, \end{cases}$$

where $a<b$ are two consecutive zeros of $u$, and $\chi:[a,b]\to\mathbb{R}$ is a continuous function,then

$$\int_a^b|\chi(t)|dt>\frac{4}{b-a} \quad (1.2)$$

Note that in order to obtain this inequality, it is supposed that $a$ and $b$ are twoconsecutivezeros of $u$. In our case, as it will be observed in the proof of our main result, we assumejust that $u$ is a nontrivial solution to $(1.1)$.

Inequality $(1.2)$ is useful in various applications, including oscillation theory, stabilitycriteria for periodic differential equations, and estimates for intervals of disconjugacy.

Several generalizations and extensions of inequality $(1.2)$ to different boundary valueproblems exist in the literature. As examples, we refer to [6-8] and the referencestherein.

Some Lyapunov-type inequalities for fractional boundary value problems have been obtained. Ferreira [14] established a fractional version of inequality $(1.2)$ for a fractionalboundary value problem involving the Riemann-Liouville fractional derivative oforder$1<\alpha\leq 2$ . More precisely, Ferreira [9] studied the fractional boundary value problem

$$\begin{cases} D^{\alpha}_{a+}(t)+\chi(t)u(t)=0, & a<t<b \\ u(a)=u(b)=0, \end{cases} \qquad (1.3)$$

where $D^{\alpha}_{q,a^+}$ is the Riemann-Liouville fractional derivative of order $1<\alpha\leq 2$ , and $\chi:[a,b]\to\mathbb{R}$ is a continuous function. In this case, it was proved that if $(1.3)$ has a nontrivialsolution, then

$$\int_a^b|\chi(t)|dt>\Gamma(\alpha)\left(\frac{4}{b-a}\right)^{\alpha-1},$$

where $\Gamma$ is the Euler gamma function. Observe that if we take $\alpha=2$ in the last inequality, we obtain the standard Lyapunov inequality $(1.2)$ .

Recently, in [10], authors research Lyapunov-type inequalities for a fractional $p$- Laplacian equation. For other related results, we refer to [11], [12] and the references therein.

The paper is organized as follows. In Section 2, we recall some basic concepts on fractional $q$- calculus and establish some preliminary results that will be used in Section 3, where we state and prove our main result. In Section 4, we present some applications of the obtained Lyapunov-type inequalities to eigenvalue problems.

## II. PRELIMINARIES

For the convenience of the reader, we recall some basic concepts on fractional $q$- calculus tomake easy the analysis of $(1.1)$ . For more details, we refer to [13].

Let $C[a,b]$ be the set of real-valued and continuous functions in $[a,b]$ . Let $f\in C[a,b]$ .Let $\alpha\geq 0$ . The Riemann-Liouville fractional $q$- integral of order $\alpha$ of $f$ is defined by $I^0_{q,a}f\equiv f$ and

$$\left(I^{\alpha}_{q,a^+}f\right)(t)=\frac{1}{\Gamma_q(\alpha)}\int_a^t(t-qs)^{(\alpha-1)}f(s)d_qs, \qquad \alpha>0, t\in[a,b],$$

where $\Gamma_q$ is the $q$- gamma function.

The $q$- derivative is defined by

$$\left(D_qf\right)(t)=\frac{f(t)-f(qt)}{t-qt}\quad(t\neq 0), \qquad \left(D_qf\right)(0)-\lim_{x\to 0}\left(D_qf\right)(t)$$

The Riemann-Liouville fractional $q$- derivative of order $\alpha\geq 0$ of $f$ is defined by

$$\left(D^{\alpha}_{q,a^+}f\right)(t)=\left(D^n_qI^{n-\alpha}_{q,a^+}f\right)(t), \qquad t\in[a,b]$$

where $n=[\alpha]+1$.

**Lemma 2.1**[13] Let $\alpha>0$ . If $D^{\alpha}_{q,a^+}u\in C[a,b],$ then

$$I^{\alpha}_{q,a^+}D^{\alpha}_{q,a^+}u(t)=u(t)+\sum_{i=1}^{n}k_i(t-a)^{(\alpha-i)}$$

where $n=[\alpha]+1$.

Now, in order to obtain an integral formulation of $(1.1)$ , we need the following results.

**Lemma 2.2** Let $2<\alpha\leq 3,$ and $y\in C[a,b]$ .Then the problem

$$\begin{cases} D^{\alpha}_{q,a^+}u(t)+y(t)=0, & a<t<b, \\ u(a)=D_qu(a)=D_qu(b)=0, \end{cases}$$

has a unique solution

$$u(t)=\int_a^b G(t,s)y(s)d_qs$$

where

$$G(t,s)=\frac{1}{\Gamma_q(\alpha)}\begin{cases}\left(\frac{b-qs}{b-a}\right)^{(\alpha-2)}(t-a)^{(\alpha-1)}-(t-qs)^{(\alpha-1)}, & a\le qs\le t\le b\\ \left(\frac{b-qs}{b-a}\right)^{(\alpha-2)}(t-a)^{(\alpha-1)}, & a\le t\le qs\le b\end{cases}$$

*Proof*From Lemma2.1. we have

$$u(t)=-\left(I_{q,a^+}^{\alpha}y\right)(t)+k_1(t-a)^{(\alpha-1)}+k_2(t-a)^{(\alpha-2)}+k_3(t-a)^{(\alpha-3)}.$$

for some real constants $k_i, i=1,2,3,$ and the condition $u(a)=0$ yields $k_3=0$ . Therefore,

$$D_qu(t)=-\left(I_{q,a^+}^{\alpha-1}y\right)(t)+k_1[\alpha-1]_q(t-a)^{(\alpha-2)}+k_2[\alpha-2]_q(t-a)^{(\alpha-3)}.$$

The condition $D_qu(a)=0$ implies that $k_2=0$ . Since $D_qu(b)=0$ , we get

$$k_1=\frac{1}{\Gamma_q(\alpha)(b-a)^{(\alpha-2)}}\int_a^b(b-qs)^{(\alpha-2)}y(s)d_qs.$$

Thus,
$$u(t)=-\int_a^t\frac{(t-qs)^{(\alpha-1)}y(s)}{\Gamma_q(\alpha)}d_qs+\int_a^b\frac{1}{\Gamma_q(\alpha)}\left(\frac{b-qs}{b-a}\right)^{(\alpha-2)}(t-a)^{(\alpha-1)}y(s)d_qs.$$

For the uniqueness, suppose that $u_1$ and $u_2$ are two solutions of the considered problem. Define $u=u_1-u_2$ . By linearity, u solves the boundary value problem

$$\begin{cases}D_{q,a^+}^{\alpha}u(t)=0, & a<t<b,\\ u(a)=D_qu(a)=D_qu(b)=0,\end{cases}$$

which has as a unique solution $u=0$ . Therefore, $u_1=u_2$ ,and the uniqueness follows.

**Lemma 2.3** Let $y\in C[a,b], 2<\alpha\le3, 1<\beta\le2, p>1$ and $\frac{1}{p}+\frac{1}{g}=1.$ Then the problem

$$\begin{cases}D_{q,a^+}^{\beta}\left(\Phi_p\left(D_{q,a^+}^{\alpha}u(t)\right)\right)+y(t)=0, & a<t<b\\ u(a)=D_qu(a)=D_qu(b)=0, & D_{q,a^+}^{\alpha}u(a)=D_{q,a^+}^{\alpha}u(b)=0\end{cases}$$

has a unique solution

$$u(t)=-\int_a^b G(t,s)\Phi_g\left(\int_a^b H(s,\tau)y(\tau)d_q\tau\right)d_qs,$$

where

$$H(t,s)=\frac{1}{\Gamma_q(\beta)}\begin{cases}\left(\frac{b-qs}{b-a}\right)^{(\beta-1)}(t-a)^{(\beta-1)}-(t-qs)^{(\beta-1)}, & a\le qs\le t\le b,\\ \left(\frac{b-qs}{b-a}\right)^{(\beta-1)}(t-a)^{(\beta-1)}, & a\le t\le qs\le b,\end{cases}$$

*Proof*From Lemma2.1 and Lemma2.2.we have

$$\Phi_p\left(D_{q,a^+}^{\alpha}u(t)\right)=-\int_a^t\frac{(t-qs)^{(\beta-1)}}{\Gamma_q(\beta)}y(s)d_qs+\frac{1}{\Gamma_q(\beta)}\int_a^b\left(\frac{b-qs}{b-a}\right)^{(\beta-1)}(t-a)^{(\beta-1)}y(s)d_qs,$$

that is,

$$\Phi_p\left(D_{q,a^+}^{\alpha}u(t)\right)=\int_a^b H(t,s)y(s)d_qs.$$

Then we have

$$D_{q,a^+}^{\alpha}u(t)-\Phi_g\left(\int_a^b H(t,s)\,y(s)\,d_qs\right)=0.$$

Setting

$$y\%(t)=-\Phi_g\left(\int_a^b H(t,s)\,y(s)\,d_qs\right),$$

we obtain

$$\begin{cases} D_{q,a^+}^{\alpha}u(t)+y\%(t)=0, & a<t<b,\\ u(a)=D_qu(a)=D_qu(b)=0. \end{cases}$$

Finally, applying Lemma2.2, we obtain the desired result.

The following estimates will be useful later.

**Lemma 2.4** We have

$$0\le G(t,s)\le G(b,s),\qquad (t,s)\in[a,b]\times[a,b].$$

*Proof* $q$- differentiating with respect to $t$ , we obtain

$$_tD_qG(t,s)=\frac{1}{\Gamma_q(\alpha-1)}\begin{cases}\left(\dfrac{b-qs}{b-a}\right)^{(\alpha-2)}(t-a)^{(\alpha-2)}-(t-qs)^{(\alpha-2)}, & a\le qs\le t\le b,\\ \left(\dfrac{b-qs}{b-a}\right)^{(\alpha-2)}(t-a)^{(\alpha-2)}, & a\le t\le qs\le b.\end{cases}$$

Set

$$g_1(t,s)=\left(\frac{b-qs}{b-a}\right)^{(\alpha-2)}(t-a)^{(\alpha-2)}-(t-qs)^{(\alpha-2)},\quad a\le qs\le t\le b.$$

and

$$g_2(t,s)=\left(\frac{b-qs}{b-a}\right)^{(\alpha-2)}(t-a)^{(\alpha-2)},\quad a\le t\le qs\le b.$$

Clearly

$$g_2(t,s)\ge 0,\quad a\le t\le qs\le b.$$

On the other hand, using the inequality

$$tb\ge asq\cdot q^{2n+\alpha-2},\quad a\le qs\le t\le b,\quad n\in\text{¥},\quad \alpha>2,\quad q\in(0,1)$$

we obtain

$$g_1(t,s)\ge 0,\quad a\le qs\le t\le b.$$

As consequence, we have

$$G(t,s)\ge 0,\qquad (t,s)\in[a,b]\times[a,b].$$

Then $G(\text{g}\,s)$ is a nondecreasing function for all $s\in[a,b]$, which yields

$$0=G(a,s)\le G(t,s)\le G(b,s),\qquad (t,s)\in[a,b]\times[a,b].$$

The proof is complete.

**Lemma 2.5** We have

$$0\le H(t,s)\le H(s,s),\qquad (t,s)\in[a,b]\times[a,b].$$

*Proof*Observe that $H(t,s)=\,_tD_qG(t,s)$ for $\alpha=\beta+1$. Then, from the proof ofLemma2.4 wehave

$$H(t,s)\ge 0,\qquad (t,s)\in[a,b]\times[a,b].$$

On the other hand, for all $s\in[a,b]$, we have

$$\Gamma_q(\beta)H(s,s)=\left(\frac{b-qs}{b-a}\right)^{(\beta-1)}(s-a)^{(\beta-1)}$$

For $a\le t\le qs\le b,$ we have

$$\Gamma_q(\beta)H(t,s)=\left(\frac{b-qs}{b-a}\right)^{(\beta-1)}(t-a)^{(\beta-1)}\le\left(\frac{b-qs}{b-a}\right)^{(\beta-1)}(s-a)^{(\beta-1)}=\Gamma_q(\beta)H(s,s).$$

For $a\le qs\le t\le b,$ we have

$$\Gamma_q(\beta)H(t,s)=\left(\frac{b-qs}{b-a}\right)^{(\beta-1)}(t-a)^{(\beta-1)}-(t-qs)^{(\beta-1)}$$

Let $s\in[a,b)$ be fixed.Define the function $\psi:(s,b]\to\mathbb{R}$ by

$$\psi(t)=\Gamma_q(\beta)H(t,s),\qquad t\in(s,b].$$

We have

$${}_tD_q\psi(t)=[\beta-1]_q\left(\left(\frac{b-qs}{b-a}\right)^{(\beta-1)}(t-a)^{(\beta-2)}-(t-qs)^{(\beta-2)}\right),\qquad t\in(s,b].$$

Using the inequalities

$$\left(\frac{b-qs}{b-a}\right)^{(\beta-1)}\le 1,\ \ \beta-2\le 0,\ \ (t-a)^{(\beta-2)}\le(t-qs)^{(\beta-2)},$$

we get

$${}_tD_q\psi(t)\le 0,\qquad t\in(s,b].$$

Thus, for all $t\in[a,b)$, we have

$$\psi(t)\le\psi(s),$$

that is,

$$\Gamma_q(\beta)H(t,s)\le\Gamma_q(\beta)H(s,s),\qquad t\in(s,b].$$

The proof is complete.

Now, we are ready to state and prove our main result.

## III. MAIN RESULT

Our main result is the following Lyapunov-type inequality.

**Theorem 3.1**Suppose that $2<\alpha\le 3,\ 1<\beta\le 2,\ p>1,$ and $\chi:[a,b]\to\mathbb{R}$ is a continuousfunction. If $(1.1)$ has a nontrivial continuous solution, then

$$\int_a^b(b-qs)^{(\beta-1)}(s-a)^{(\beta-1)}|\chi(s)|d_qs$$
$$\ge\Gamma_q(\beta)\left[\Gamma_q(\alpha)\right]^{p-1}(b-a)^{(\beta-1)}\left(\int_a^b(b-qs)^{(\alpha-2)}(qs-a)d_qs\right)^{1-p}.\ (3.1)$$

*Proof*We endow the set $C[a,b]$ with the Chebyshev norm $\|g\|_\infty$ given by

$$\|u\|_\infty=\max\left\{|u(t)|:a\le t\le b\right\},\qquad u\in C[a,b].$$

Suppose that $u\in C[a,b]$ is a nontrivial solution of $(1.1)$. From Lemma2.3we have

$$u(t)=-\int_a^bG(t,s)\Phi_g\left(\int_a^bH(s,\tau)\chi(\tau)\Phi_p(u(\tau))d_q\tau\right)d_qs,\qquad t\in[a,b].$$

Let $t\in[a,b]$ be fixed.We have

$$|u(t)|\le\int_a^b|G(t,s)|\left|\int_a^bH(s,\tau)\chi(\tau)\Phi_p(u(\tau))d_q\tau\right|^{g-1}d_qs$$

$$\le\int_a^b|G(t,s)|\theta(s)d_qs,$$

Where $\theta(s)=\left|\int_a^b|H(s,\tau)||\chi(\tau)||u(\tau)|^{p-1}d_q\tau\right|^{g-1},\qquad s\in[a,b].$

Using Lemma2.4and Lemma2.5, we obtain

$$\left|u(t)\right| \le \left\|u\right\|_\infty^{(p-1)(g-1)} \left(\int_a^b G(b,s)\,d_q s\right)\left(\int_a^b H(s,s)\left|\chi(s)\right| d_q s\right)^{g-1}.$$

Since the last inequality holds for every $t \in [a,b]$, we obtain

$$1 \le \left(\int_a^b G(b,s)\,d_q s\right)\left(\int_a^b H(s,s)\left|\chi(s)\right| d_q s\right)^{g-1},$$

which yields the desired result.

**Corollary3.2** Suppose that $2 < \alpha \le 3,\ 1 < \beta \le 2,\ p > 1,$ and $\chi : [a,b] \to \mathbb{R}$ is a continuousfunction. If $(1.1)$ has a nontrivial continuous solution, then

$$\int_a^b \left|\chi(s)\right| d_q s \ge \frac{\left[\Gamma_q(\alpha)\right]^{p-1} \Gamma_q(\beta)\left(q^{\beta-1}+1\right)^{2\beta-2}}{q^{\beta-1}\left((b-aq)+aq^{\beta-1}(1-q)\right)^{(\beta-1)}} \left(\int_a^b (b-qs)^{(\alpha-2)}(qs-a)\,d_q s\right)^{1-p}. \quad (3.2)$$

*Proof*Let

$$\psi(s) = (b-qs)^{(\beta-1)}(s-a)^{(\beta-1)}, \qquad s \in [a,b].$$

We have

$$D_q\psi(s) = \frac{[\beta\text{-}1]_q (bs)^{\beta-2}\left(\frac{q^2 s}{b};q\right)_\infty \left(\frac{a}{s};q\right)_\infty}{\left(q^{\beta-1}\frac{q^2 s}{b};q\right)_\infty \left(q^{\beta-1}\frac{a}{s};q\right)_\infty}\left(\frac{b}{1-\frac{a}{s}q^{\beta-2}} - \frac{sq}{1-\frac{q^2 s}{b}q^{\beta-2}}\right)$$

Observe that the function $\psi$ has a maximum at the point $D_q\psi(s)=0$, that is,

$$s = \frac{b+aq^{\beta-1}}{q\left(q^{\beta-1}+1\right)}. \text{ So } \left\|\psi\right\|_\infty = \frac{q^{\beta-1}(b-a)^{(\beta-1)}}{\left(q^{\beta-1}+1\right)^{2\beta-2}}\left((b-aq)+aq^{\beta-1}(1-q)\right)^{(\beta-1)}.$$

The desired result follows immediately from the last equality and inequality $(3.1)$.

For $p = 2$, problem $(1.1)$ becomes

$$\begin{cases} D_{q,a^+}^{\beta}\left(D_{q,a^+}^{\alpha} u(t)\right) + \chi(t)u(t) = 0, \quad a < t < b, \\ u(a) = D_q u(a) = D_q u(b) = 0, \quad D_{q,a^+}^{\alpha} u(a) = D_{q,a^+}^{\alpha} u(b) = 0, \end{cases} \quad (3.3)$$

where $2 < \alpha \le 3,\ 1 < \beta \le 2,\ p > 1,$ and $\chi : [a,b] \to \mathbb{R}$ is a continuousfunction. In this case, taking $p = 2$, in Theorem3.1, we obtain the following result.

**Corollary3.3** Suppose that $2 < \alpha \le 3,\ 1 < \beta \le 2,\ p > 1,$ and $\chi : [a,b] \to \mathbb{R}$ is a continuousfunction. If $(3.3)$ has a nontrivial continuous solution, then

$$\int_a^b (b-qs)^{(\beta-1)}(s-a)^{(\beta-1)}\left|\chi(s)\right| d_q s$$

$$\ge \Gamma_q(\beta)\Gamma_q(\alpha)(b-a)^{(\beta-1)}\left(\int_a^b (b-qs)^{(\alpha-2)}(qs-a)\,d_q s\right)^{-1}.$$

Taking $p = 2$, in Corollary3.2, we obtain the following result.

**Corollary3.4** Suppose that $2 < \alpha \le 3,\ 1 < \beta \le 2,\ p > 1,$ and $\chi : [a,b] \to \mathbb{R}$ is acontinuousfunction. If $(3.3)$ has a nontrivial continuous solution, then

$$\int_a^b \left|\chi(s)\right| d_q s \ge \frac{\Gamma_q(\alpha)\Gamma_q(\beta)\left(q^{\beta-1}+1\right)^{2\beta-2}}{q^{\beta-1}\left((b-aq)+aq^{\beta-1}(1-q)\right)^{(\beta-1)}}\left(\int_a^b (b-qs)^{(\alpha-2)}(qs-a)\,d_q s\right)^{-1}.$$

## IV. APPLICATIONS TO EIGENVALUE PROBLEMS

In this section, we present some applications of the obtained results to eigenvalue problems.

**Corollary4.1** Let $\lambda$ be an eigenvalue of the problem

$$\begin{cases} D^{\beta}_{q,0^+}\left(\Phi_p\left(D^{\alpha}_{q,0^+}u(t)\right)\right)+\lambda\Phi_p u(t)=0, & 0<t<1, \\ u(0)=D_q u(0)=D_q u(1)=0, & D^{\alpha}_{q,a^+}u(0)=D^{\alpha}_{q,a^+}u(1)=0, \end{cases} \quad (4.1)$$

where $2<\alpha\leq 3,\ 1<\beta\leq 2,$ and $p>1,$ then

$$|\lambda|\geq\frac{\Gamma_q(2\beta)}{\Gamma_q(\beta)}\left(\frac{q\Gamma_q(\alpha)\Gamma_q(\alpha+1)}{\Gamma_q(\alpha-1)}\right)^{(p-1)}. \quad (4.2)$$

*Proof*Let $\lambda$ be an eigenvalue of $(4.1)$. Then there exists a nontrivial solution $u=u_\lambda$ to $(4.1)$.Using Theorem3.1 with $(a,b)=(0,1)$ and $\chi(s)=\lambda$, we obtain

$$|\lambda|\int_0^1(1-qs)^{(\beta-1)}s^{(\beta-1)}d_qs\geq\Gamma_q(\beta)\left[\Gamma_q(\alpha)\right]^{p-1}\left(\int_0^1 qs(1-qs)^{(\alpha-2)}d_qs\right)^{1-p}.$$

Observe that

$$\int_0^1(1-qs)^{(\beta-1)}s^{(\beta-1)}d_qs=B_q(\beta,\beta)$$

and $\int_0^1 qs(1-qs)^{(\alpha-2)}d_qs=q\int_0^1 s^{2-1}(1-qs)^{(\alpha-1)-1}d_qs=qB_q(2,\alpha-1),$

where $B_q$ is the beta function defined by

$$B_q(s,t)=\int_0^1 u^{(s-1)}(1-qu)^{(t-1)}d_qu,\qquad s,t>0.$$

Using the identity

$$B_q(s,t)=\frac{\Gamma_q(s)\Gamma_q(t)}{\Gamma_q(s+t)},$$

we get the desired result.

**Corollary4.2** Let $\lambda$ be an eigenvalue of the problem

$$\begin{cases} D^{\beta}_{q,0^+}\left(D^{\alpha}_{q,0^+}u(t)\right)+\lambda u(t)=0, & 0<t<1, \\ u(0)=D_q u(0)=D_q u(1)=0, & D^{\alpha}_{q,a^+}u(0)=D^{\alpha}_{q,a^+}u(1)=0, \end{cases}$$

where $2<\alpha\leq 3,\ 1<\beta\leq 2,$ and $p>1,$ then

$$|\lambda|\geq\frac{q\Gamma_q(\alpha)\Gamma_q(\alpha+1)\Gamma_q(2\beta)}{\Gamma_q(\alpha-1)\Gamma_q(\beta)}. \quad (4.3)$$

*Proof*It follows from inequality $(4.2)$ by taking $p=2$.

## REFERENCES

[1]. Bognàr, G, Rontó, M: Numerical-analytic investigation of the radially symmetric solutions for some nonlinear PDEs.Comput. Math. Appl. 50, 983-991 (2005)

[2]. Glowinski, R, Rappaz, J: Approximation of a nonlinear elliptic problem arising in anonNew- tonian fluid flow model in glaciology. Math. Model. Numer. Anal. 37(1), 175-186 (2003)

[3]. Kawohl, B: On a family of torsional creep problems. J. Reine Angew. Math. 410, 1-22 (1990)

[4]. Liu, C: Weak solutions for a viscous p-Laplacian equation.Electron.J.Differ.Equ.2003,63(2003)

[5]. Liapunov, AM: Problème général de la stabilité dumouvement.Ann.Fac.Sci.Univ.Toulouse2, 203-407 (1907)

[6]. Aktas, MF: Lyapunov-type inequalities for a certain class of n-dimensional quasilinearsystems. Electron. J. Differ. Equ.2013, 67 (2013)

[7]. Hartman, P, Wintner, A: On an oscillation criterion of Liapounoff.Am.J.Math.73,885-890(1951)

[8]. Pachpatte, BG: On Lyapunov-type inequalities for certain higher order differential equations. J. Math. Anal. Appl. 195,527-536 (1995)
[9]. Ferreira, RAC: A Lyapunov-type inequality for a fractional boundary value problem. Fract.Calc. Appl. Anal. 16(4),978-984 (2013)
[10]. Arifi, Nassir Al, et al. "Lyapunov-type inequalities for a fractional p -Laplacian equation."
[11]. Journal of Inequalities & Applications2016.1(2016):189.
[12]. Jleli, M, Samet, B: A Lyapunov-type inequality for a fractional q-difference boundary value problem. J. Nonlinear Sci.Appl. 9, 1965-1976 (2016)
[13]. O'Regan, D, Samet, B: Lyapunov-type inequalities for a class of fractional differentialequations. J. Inequal. Appl. 2015,247 (2015)
[14]. Rajkovi′c, PM, Marinkovi′c, SD, Stankovi′c, MS: Fractional integrals and derivatives in q-calculus. Appl. Anal. DiscreteMath. 1(1), 311-323 (2007)